ZONAL FLOW AND MAGNETIC FIELD GENERATION IN THE IONOSPHERE
ON THE BASIS OF MULTISCALE EXPANSION

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Abstract

In the present work, the generation of large-scale zonal flows and magnetic field by short-scale collisionless electron skin depth order drift-Alfven turbulence in the ionosphere is investigated. The self-consistent system of two model nonlinear equations, describing the dynamics of wave structures with characteristic scales till to the skin value, is obtained. Evolution equations for the shear flows and the magnetic field is obtained by means of the averaging of model equations for the fast-high-frequency and small-scale fluctuations on the basis of multi-scale expansion. It is shown that the large-scale disturbances of plasma motion and magnetic field are spontaneously generated by small-scale drift-Alfven wave turbulence through the nonlinear action of the stresses of Reynolds and Maxwell. Positive feedback in the system is achieved via modulation of the skin size drift-Alfven waves by the large-scale zonal flow and/or by the excited large-scale magnetic field. As a result, the propagation of small-scale wave packets in the ionospheric medium is accompanied by low-frequency, long-wave disturbances generated by parametric instability. Two regimes of this instability, resonance kinetic and hydrodynamic ones, are studied. The increments of the corresponding instabilities are also found. The conditions for the instability development and possibility of the generation of large-scale structures are determined. The nonlinear increment of this interaction substantially depends on the wave vector of Alfven pumping and on the characteristic scale of the generated zonal structures. This means that the instability pumps the energy of primarily small-scale Alfven waves into that of the large-scale zonal structures which is typical for an inverse turbulent cascade. The increment of energy pumping into the large-scale region noticeably depends also on the width of the pumping wave spectrum and with an increase of the width of the initial wave spectrum the instability can be suppressed.

Key Words: Skin-size perturbations; Zonal flow; large scale magnetic field; pumping of energy with respect to scales.

1. Introduction

In recent years, special attention has been paid to the study of the generation of large-scale spatial-inhomogeneous (shear) zonal flows and magnetic field turbulence in the magnetized plasma medium in laboratory devices, as well as in space conditions (Diamond, et al., 2005). Such interest firstly is caused by the fact that the excitation of the zonal flows and large-scale magnetic field generation can lead to noticeable weakening of anomalous processes, stipulated by relatively small-scale turbulence and by passage to the modes with improved property of

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adapation to the equilibrium state (Diamond, et al., 2005; Kamide and Chian, 2007). Zonal flows are the integral parts of the collective activity of the majority of the planetary atmospheres and are manifested in the form of the large-scale low-frequency modes, propagating along the parallels (Busse, 1994; Aubert, et al., 2002). The possibility of such generation is intensively studied via some of the basic modes of the turbulence. At the present time, a question about the generation of zonal modes is mostly studied by the electrostatic drift, relatively long-wave modes, characteristic transverse wavelength of which is greater than a Larmor radius of ions according to the electron temperature (Smolyakov, et al., 2000; Shukla and Stenflo, 2002) and by some other electrostatic modes (Mikhailovskii, et al., 2006).

The previous authors made the trials of investigations of the special features of the zonal flow generation by means of drift-Alfven type fluctuation on the basis of three sufficiently simplified models, describing nonlinear interaction between these modes: the first, a class of the models in which the effect of the ion temperature is negligible and only the effect of the so-called finite Larmor radius of ions according to the electron temperature (Guzdar, et al., 2001; Lakhin, 2003) is taken into account; the second model, where both disturbances, the primary small-scale as well as the large-scale zonal disturbances, have characteristic scale less than a Larmor radius of ions \( \rho_i \) (Smolyakov, et al., 2002); and the third class of the models, where finite Larmour radius of ions are considered neglecting the skin size inertial effects (Lakhin, 2004; Mikhailovskii, et al., 2006; Shukla, 2005). Although in the work (Pokhotelov, et al., 2003), generation of the zonal flow was studied by inertia Alfvén fluctuations. But, it was made in uniform plasma neglecting finiteness of a Larmor radius of electrons, ions (\( T_e, T_i \rightarrow 0 \)).

One of the important wave modes in non-uniform magnetized space (Stasiewicz, et al., 2000; Sahraoui, et al., 2002; Narita, et al., 2007) as well as in laboratory (Gekelman, 1999; Mikhailovskii, 1978) plasma media are electromagnetic small-scale drift-Alfven (SSDA) modes with the transverse wavelengths, small in comparison with a Larmor radius of ions, \( k_\| \rho_i \gg 1 \), where \( \rho_i = (T_i / m_i \omega_{Bi})^{1/2} \) is a Larmor radius of ions, \( k_\| \) - transverse (according to external equilibrium magnetic field \( B_0 \)) wave number, \( \omega_{Bi} = eB_0 / m_i c \) - ion-cyclotron frequency, \( T_\alpha \) - the temperature of ions with \( \alpha = i \) and electrons with \( \alpha = e \), respectively, \( e \) - elementary ion charge, \( m_\alpha \) - the mass of ion with \( \alpha = i \) and electron with \( \alpha = e \), respectively, \( c \) is the speed of light. These small scale fluctuations can generate large-scale zonal modes as in the space and as in the laboratory plasma. Moreover, the contemporary theory of anomalous transfers (Kadomtsev, Pogutse, 1984; Aburjania, 2006; Aburjania, 1990) predicts, that the anomalous thermal conductivity and diffusion in the plasma medium may be stipulated, in essence, by the processes with the characteristic wavelength \( \lambda_\| \) of the order of collision-less skin length \( \lambda_s \), 

\[
\lambda_\| = 2\pi / k_\| \sim \lambda_s = c / \omega_{pe},
\]

where \( \omega_{pe} = (4\pi e^2 n_0 / m_e)^{1/2} \) is a plasma frequency. In this connection, description of the nonlinear wave processes on the scales \( \lambda_s \sim \lambda_\| < \rho_i \) appears necessary. Therefore, elaboration of the self-consistent system of nonlinear equations, describing the dynamics of SSDA wave processes, with the characteristic scales till to the skin size, i.e., taking into account a finite Larmor radius of ions and inertia processes, represents one of the goals of this work. Further, on the basis of these dynamic equations, an investigation of the special features of the nonlinear development of collective activity in the ionosphere medium on SSDA modes is important.

At present, there prevails the point of view, according to which, the spontaneous generation of large-scale zonal modes (or convective cells) are the result of the secondary instability of

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plasma fluctuations (Diamond, et al., 2005). At the basis of instability, there lies the nonlinear interaction of the primary fluctuations (pumping of one of the types of relatively short-wave drift waves, swinging by some known linear or nonlinear mechanisms), which results in the zonal flow generation. Positive feedback is ensured by modulation of the amplitudes of primary plasma fluctuations by secondary shear zonal mode, and instability can be related to the class of parametric (or modulation) instabilities. The generation of such large-scale (in comparison with the small-scale primary modes) structures can substantially increase energy transfer via medium particles.

According to investigation methods of the above-mentioned nonlinear processes, the works already existing in this direction can be divided into two groups. To the first group can be attributed the works which are based on ideas and methods of the classical theory of coherent parametric instabilities (Oraevskii, 1984) and frequently called the “parametric” approach. In them, the interaction processes of the finite number of waves are examined: pumping waves; the shear flows (wave with the low, sometimes with zero frequency) and one or two satellites of the pumping wave (Sagdeev, et al., 1978; Guzdar, et al., 2001; Smolyakov, et al., 2002; Mikhailovskii, et al., 2006). The second and alternative group includes the works (Smolyakov, et al., 2000; Lakhin, 2003; Lakhin, 2004) in which there lies an assumption about the separation of the scales of turbulence into small scale and the zonal flow (large scale), developed at the time in the work (Vedenov, Rudakov, 1964). In this approach, the small-scale turbulence is described by the wave kinetic equation, in which the influence of zone flow is considered. In the work (Smolyakov, et al., 2002), it is shown that given the initial approximations the above mentioned approaches lead to the identical results.

In this work (and also in the work (Aburjania, et al., 2008), further named Part II), for investigation of the zonal flow generation by means of skin scale SSDA fluctuations in the ionosphere plasma, we use the “parametric” approach, which, as already is mentioned above, goes back to the method used in the theory of convective cell generation (Sagdeev, et al., 1978). The method of this approach has been improved in recent works (Mikhailovskii, et al., 2006; Mikhailovskii, et al., 2006) in the sense that, instead of the separate monochromatic packet of primary modes, the spectrum of these modes with arbitrary width is investigated. In our opinion, this approach is more visual and more adequate for this problem. Consequently, this work is organized as follows. Initial nonlinear equations for our task are represented in Section 2. There on the basis of the analysis of the linear stage of disturbance propagation, we determine frequency spectrum of those investigated by us SSDA skin scale pumping waves. In Section 3 we introduce the excited values, which characterize the primary small-scale modes (pumping), secondary small-scale modes (satellites) and the zonal flows. Further in Section 3 initial equations for amplitudes of the pumping waves, satellites and the zonal modes are formulated. Here, also, the solution of these equations is conducted and expressions for the amplitudes are determined. Dispersion equation for the large-scale zonal flow and the magnetic field for the arbitrary continuous spectrum of pumping are obtained in Section 4. The analysis of this equation for the monochromatic small-scale (of order of a skin size) and relatively large-scale waves, and also for the different practically important spectra of the pumping waves is carried out in Section 1 of Part II. Here, the correspondent growth increments and the criterion of zonal flow generation are determined. Section 2 of Part II is concerned with investigating the influence of the non-monochromaticity of different pumping waves on the zonal flow.

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instability development. Finally, in Section 3 of Part II, the basic results of this investigation are assembled.

2. Initial dynamic equations

The equilibrium state of the ionosphere plasma we characterize with electron density \( n_{e0} \), the single-charged ions \( n_{i0} \), non-uniform along the axis \( x \) (\( \nabla n_{j0} \parallel x \), \( j = e, i \)), uniform temperature of the electrons \( T_e \) and the ions \( T_i \) (\( \nabla T_e \cdot \nabla T_i = 0; T_e \geq T_i \)). Non-uniform equilibrium density \( n_0(x) = n_{e0}(x) = n_{i0}(x) \) is supported by external sources (for example, external electric field, volumetric forces and others). Equilibrium magnetic field \( B_0 \) we consider uniform and directed along the \( z \) axis, \( (B_0 \parallel z) \).

For electron’s description let’s use a electron continuity equation in a drift approximation:

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (V_e n_e) - \frac{1}{cB_0} (B \cdot \nabla) J_B = 0,
\]

(1)

where \( n_e = n_0 + n_e^\prime \), \( n_0, n_e^\prime \) - equilibrium and perturbed parts of the electron density, \( V_e = (c/B_0) (e_x \times \nabla \phi) \) - drift velocity in the twisted fields, \( \phi \) - electrostatic potential, \( B = B_0 + (\nabla A \times e_z) \) - full magnetic field, \( J_B \) - current density component along the equilibrium magnetic field, \( A \) - vector potential along axis \( oz \), \( e_z \) - unit vector along the equilibrium magnetic field, \( c \) - light speed; but perturbations’ electric field \( E \) stress denote by the following relation:

\[
E = -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t} e_z,
\]

(2)

and by the longitudinal component of the electron motion equation:

\[
n_e m \frac{dV_\parallel}{dt} = -T_e \nabla \cdot \frac{\partial n_e}{\partial z} - e_n E_\parallel,
\]

(3)

where \( d/\partial t = \partial/\partial t + (c/B_0) (\nabla \phi \times \nabla)_z \), \( m \) - electron mass, \( T_e \) - electrons’ equilibrium temperature, which is thought to be constant, e-ion charge.

As a size of the investigated waves rather small or of the order of \( \rho_i \), then for ions Boltzman’s distribution is true:

\[
n_i = n_0 (1 + \kappa_n x - e\phi / T_i),
\]

(4)

where \( \kappa_n = d \ln n_0(x) / dx \), \( T_i \) - equilibrium constant ion temperature, but ions’ and electrons’ perturbed temperature is neglected.

Let’s use quasi neutrality condition, substituting (4) in (1), (3), we got the equation of small scale drift – Alfven (SSDA) waves:

\[
\frac{\partial A}{\partial t} + V_{re} \frac{\partial A}{\partial y} + c(1 + \tau) \nabla A \phi - \lambda^2 \frac{d}{dt} \Delta A = 0,
\]

(5)

\[
\frac{d}{dt} \phi + V_{re} \frac{\partial \phi}{\partial y} - \frac{V^2_{re}}{c^2} \lambda^2 \nabla A = 0.
\]

(6)

Here \( V_{re,i} = \mp cT_{e,i} \kappa_n / (eB_0) \), \( V_{Te} = (T_e / m_e)^{1/2} \) - electrons’ thermal velocity; \( \tau = T_e / T_i \), \( \nabla_\parallel = \partial / \partial z - B_0^{-1} (\nabla A \times \nabla)_z \).

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Getting (5), (6) ion longitudinal motion is neglected and it is supposed that longitudinal current $J_\parallel$ are caused by plasma electrons, $J_\parallel = -c\Delta x A / 4\pi$.

The equations (3), (4) conserve an energy integral:

$$E = \frac{1}{2} \left[ (\nabla A)^2 + \lambda_\perp^2 (\Delta A)^2 + \frac{c}{V_T} \lambda_\perp \tau (1 + \tau) \phi^2 \right] dr.$$  \hspace{1cm} (7)

Thus, system of nonlinear equations with partial derivatives (5)-(7) describe nonlinear dynamics of the skin size drift-Alfven waves in magnetically active (ionospheric, magnetospheric, laboratory) plasma. The equations (5) and (6) contain two sources of zonal mode generation: first – nonlinear terms $(\nabla \phi \times \nabla)_x \Delta \perp A$ and $(\nabla A \times \nabla)_x \phi$ (containing $\phi$), causing quasi electrostatic Reinold’s stress, and second – nonlinear term $(\nabla A \times \nabla)_x \Delta \perp A$ (containing $A$), causing electromagnetic Maxwell’s stress.

We use the system (5)-(7) for a theoretical investigation of the features of the energy pumping from small scale drift-Alfven perturbations to the large scale zonal flows and to the large-scale magnetic fields in the ionosphere medium.

### 2.1. Spectra of the linear waves

Linearising the equations (5), (6), for plane waves $\sim \exp[i(kr - \omega_k t)]$, we get the dispersion relation

$$(\omega_k - \omega_i) \left[ \omega_k (1 + k_\perp^2 \lambda_\perp^2) - \omega_e \right] - k_\perp^2 V_A^2 k_\parallel^2 \rho_i^2 (1 + \tau) = 0,$$ \hspace{1cm} (8)

Here $\omega_{i,e,i} = k_y V_{i,e,i}$ – ion and electron drift frequencies, $k_\perp = (k_x^2 + k_y^2)^{1/2}$, $k_\parallel$ – transversal and longitudinal (according to external magnetic field $B_0$) wave vectors of the perturbations, $V_A = B_0 / \sqrt{4\pi n_0 m_i}$ – Alfven velocity. The equation (8) describes interrelation of the kinetic Alfven waves and the drift waves in non uniform space plasma. Neglecting the drift effects ($\omega >> \omega_{i,e,i}$) the equations transforms into following relation:

$$\omega_k^2 = \omega_{i,e,i}^2 / (1 + k_\perp^2 \lambda_\perp^2),$$ \hspace{1cm} (9)

where $\omega_{i,e,i}^2 = (1 + \tau)(k_x V_A k_\perp \rho_i)^2$ – square of the frequency of the kinetic Alfven waves. In electrostatic limits ($k_\perp \to 0$) the dispersion relation (8) describes ion and electron drift waves:

$$\omega_{1k} = \omega_{i,i} \hspace{1cm} \omega_{2k} = \omega_{e,i} / (1 + k_\perp^2 \lambda_\perp^2).$$ \hspace{1cm} (10)

In case of the large scale perturbations, $k_\perp = k_\parallel = 0$, from (8), the following solution $\omega = 0$ will be obtained, which corresponds to the zonal flow or to zonal magnetic field. Such mode with zero frequency is damping periodically, when in the medium the dissipative processes (friction, viscosity) are present. It will be shown below that during small-scale turbulence the zonal flows and the large scale magnetic field generation process can become unstable due to turbulent feeding.

It must be mentioned, that the generation-twisting of the Alfven perturbations in the linear stage in the ionosphere or in magnetosphere is possible. This can occur in three scenarios: due to dissipative instability, caused by effective medium viscosity growth during scattering of the high frequency waves on the particles (Mikhailovskii, Pokhotelov, 1975); due to low frequency modulation instability caused by the beating of two external high frequency electromagnetic waves (Aburjania, 2007); and also due to temperature-anisotropic (mirror) instability (Treumann, et al., 2004).

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3. The equation of interrelated modes

3.1. Three-wave representations of the perturbations

In the initial plasma-dynamical equations (5), (6) the nonlinear terms contained in the expressions of operators \(d/dt\) and \(\nabla_1\), cause interrelation between the different modes. We consider three-wave scenarios of mode interrelation, at which an interaction between the SSDA pumping modes (initial modes) and their satellites (secondary SS modes) generates the large scale low frequency modes, e.i. zonal flows. Correspondingly, we represent each perturbed value \(X = (A, \phi)\) in equations (5), (6) as a sum of the three components:

\[
X = \tilde{X} + \hat{X} + \bar{X},
\]

where

\[
\tilde{X} = \sum_k \left[ \tilde{X}_+(k) \exp(ik \cdot r - i\omega_k t) + \tilde{X}_-(k) \exp(ik \cdot r + i\omega_k t) \right],
\]

describes a spectrum of SSDA pumping modes (initial), \(k = (k_x, k_y, k_z)\), \(\omega\)-wave vector and frequency of the initial modes, amplitude satisfies the condition \(\tilde{X}_- = \tilde{X}_+^*\), where asterisk indicates a complex conjugation

\[
\hat{X} = \sum_k \left[ \hat{X}_+(k) \exp(ik \cdot r - i\omega_k t) + \hat{X}_-(k) \exp(ik \cdot r + i\omega_k t) + \text{c.c.} \right],
\]

describes the small scale satellite (secondary) modes and

\[
\bar{X} = \bar{X}_0 \exp(-i\Omega t + iq_x x) + \text{c.c.},
\]

describes zonal flows. Laws of energy and impulse conservation is written in the next form: \(\omega_\pm = \Omega \pm \omega_k\) and \(k_\pm = q_x e_x + k\), respectively. Thus, the pairs \((\omega_k, k)\) and \((\Omega, q_x e_x)\) represent frequency and wave vector of SSDA pumping modes and zonal flows, respectively. Amplitude of the zonal modes \(\bar{X}_0 = (\bar{A}_0, \bar{\phi}_0)\) is considered to be constant in local approach. Further analyze will be carried out in the frames of the standard approximation \(q_x/k_\perp << 1\), \(\Omega/\omega << 1\).

3.2. Equations for amplitude of high frequency initial pumping waves

Following the standard quasi nonlinear procedure, we substitute the expressions (12)-(14) in the equations (5), (6) and neglect small nonlinear terms, connected with high frequency modes. Consequently, equalizing the coefficients of the same harmonic functions, we got equation for amplitude of initial high frequency modes:

\[
(\omega_k - \omega_\pm) \tilde{\phi}_\pm(k) - \frac{cT_k k^2}{4\pi e_n} \tilde{A}_\pm(k) = 0,
\]

\[
k_x c(1 + \tau) \tilde{\phi}_\pm(k) - \left[ \omega_k (1 + k_\perp^2 \lambda_s^2) \right] \tilde{A}_\pm(k) = 0.
\]

Let’s mention, that the condition of non trivial solution of this homogeneous system gives the dispersion relation of SSDA initial modes, coinciding with the equation (8).

3.2. Equation for amplitude of secondary small scale modes

Analogously, from the equation (5), (6), by means of (11)-(14), for amplitude \(\hat{A}_\pm\) and \(\hat{\phi}_\pm\) we got the equation:

\[
(\omega_\pm + \omega_\pm) \hat{\phi}_\pm(k) + \frac{cT_k k^2}{4\pi e_n} \hat{A}_\pm(k) = \hat{\alpha}_\pm \frac{cT_k k^2}{4\pi e_n} \left( k^2 - q^2 \right),
\]

\[
\mp k_x c(1 + \tau) \hat{\phi}_\pm + \left[ (1 + k_\perp^2 \lambda_s^2) \omega_\pm \mp \omega_\pm \right] \hat{A}_\pm = \mp \alpha_0 \frac{\bar{\Omega}_0 (1 + k_\perp^2 \lambda_s^2) \omega_k - \omega_\pm}{A_0 (1 + k_\perp^2 \lambda_s^2) \omega_k - \omega_\pm}.
\]

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Here
\[
\alpha_0^\pm = \frac{1 + \tau + k_1^2 \lambda_s^2}{1 + \tau + q_k^2 \lambda_s^2}, \quad \alpha_1^\pm = \frac{ic}{B_0} k_y q_x (l + \tau) \vec{A}_0 \phi_+,
\]
\[
\alpha_2^\pm = \frac{ic}{B_0} k_y q_x (l + \tau + q_k^2 \lambda_s^2) \vec{A}_0 \phi_+.
\]

Solution of the equation (17), (18) has the form:
\[
\hat{\phi}_\pm = \mp \frac{1}{D_\pm} \frac{ck_z T_i}{4\pi c^2 n_0} \left[ \alpha_2^\pm \left( \frac{\omega_0}{A_0} + \frac{c k_z (l + \tau)}{\omega_0 - \omega_c} \right) + \alpha_1^\pm \frac{\epsilon k_z (l + \tau)}{\omega_0 - \omega_c} \right] + \alpha_2^\pm \frac{c^2 k_z^2 T_i}{4\pi c^2 n_0} \left( k_1^2 - q_k^2 \right) (l + \tau),
\]
\[
\hat{A}_\pm = \mp \frac{1}{D_\pm} \left[ \alpha_1^\pm (\omega_0 \mp \omega_s) \left( 1 - \alpha_0 \frac{\phi_0}{A_0} \right) \frac{ck_z (l + \tau)}{\omega_0 - \omega_c} \right] \pm \alpha_2^\pm \frac{c^2 k_z^2 T_i}{4\pi c^2 n_0} \left( k_1^2 - q_k^2 \right) (l + \tau).
\]

Here
\[
D_\pm = (\Omega \pm \omega_k \mp \omega_s) \left[ (1 + k_1^2 \lambda_s^2)(\Omega \pm \omega_k) \mp \omega_s \right] - \left( 1 + \frac{T_i}{T_i} \right) k_z^2 V_\lambda^2 k_1^2 \rho_i^2.
\]

From (19)-(21) it is clear, that resulting from interaction with the large scale zonal flows and the magnetic fields, amplitudes of the secondary fast small scale perturbations depend not only on amplitudes and spatial-temporal characteristics of the fast initial (15), (16) perturbations but also on amplitudes and spatial temporal characteristics of the slow large scale zonal flows and magnetic fields ($A_0, \phi_0; \Omega, q_s$).

### 3.3. Equation for amplitude of large-scale modes of zonal flows and magnetic fields

Equations for amplitudes of the large scale zonal modes can be got substituting an expression (11)-(14) into equations (5), (6) and averaging the obtained equations according to the fast small-scale oscillations:
\[
i \Omega \vec{\phi}_0 = -\frac{c T_i q_z^2}{4\pi c^2 n_0 B_0} \sum_k k_y R_1(k),
\]
\[
- i \Omega (l + q_k^2 \lambda_s^2) \vec{A}_0 = \frac{c q_k (l + \tau)}{B_0} \sum_k k_y R_2(k) + \frac{c}{B_0} q_k \lambda_s^2 \sum_k k_y R_3(k).
\]

Here
\[
R_1(k) = q_x \left( \vec{A}_- \vec{A}_+ - \vec{A}_+ \vec{A}_- \right) + 2k_x \left( \vec{A}_- \vec{A}_+ - \vec{A}_+ \vec{A}_- \right),
\]
\[
R_2 = \vec{\phi}_- \vec{\delta}_+ - \vec{\phi}_+ \vec{\delta}_- + \vec{\phi}_- \vec{\delta}_+, \quad \vec{\delta}_\pm = \vec{A}_\pm - \frac{ck_z (l + \tau)}{l + k_1^2 \lambda_s^2} \omega_k - \omega_c \vec{\phi}_+,\n\]
\[
R_3 = k_z^2 \left( \vec{A}_- \vec{\phi}_+ + \vec{\phi}_- \vec{A}_+ - \vec{A}_- \vec{\phi}_+ - \vec{\phi}_- \vec{A}_+ \right) + q_k^2 \left( \vec{\phi}_- \vec{A}_+ - \vec{\phi}_+ \vec{A}_- \right) + 2q_k k_x \left( \vec{\phi}_- \vec{A}_+ + \vec{\phi}_+ \vec{A}_- \right).
\]

Now, it's very easy to transform the expression for the coefficient $\hat{\delta}_\pm$ to the form below by means of solutions for amplitudes of the satellites (20), (21):

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\[
\hat{\delta}_\pm = \mp \frac{1}{D_\pm} \left[ \alpha_i^\pm \left( 1 - \alpha_0 \bar{\phi}_0 \frac{ck_z (1 + \tau)}{A_0 \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega} \right) \right] \left[ \Omega - \frac{k_x q_x (\omega_k - \omega_\pi)}{k^2} (2 \pm q_x) \right] - \\
\frac{\alpha_i^\pm \Omega}{\left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega}.
\]  

(28)

In this relation, we neglect the terms of the order \( \Omega (q_x / k_x)^2 \), e.i. their contribution is not sufficient for the given problem.

Taking into account that \( \Omega \) and \( q_x \) are the small parameters, expression for \( D_\pm \) can be presented via decomposition:

\[
D_\pm = \pm D^{(0)} + D^{(1)},
\]  

(29)

Where

\[
D^{(0)} = \Omega \left[ \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_\omega - \omega_\pi \right] - \frac{2q_x k_x}{k^2} (\omega_k - \omega_\pi) \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega.
\]  

(30)

\[
D^{(1)} = \Omega \left( 1 + k_\perp^2 \lambda_s^2 \right) - \frac{q_x^2}{k^2} (\omega_k - \omega_\pi) \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega.
\]  

(31)

The expressions (20), (21) for amplitudes of the secondary small scale modes can be presented analogously:

\[
\hat{A}_\pm = \hat{A}^{(0)}_\pm + \hat{A}^{(1)}_\pm,
\]  

(32)

where

\[
\hat{A}^{(0)}_\pm = \mp \frac{(\omega_k - \omega_\omega)}{D^{(0)}} \left[ \alpha_i^\pm + \alpha_2^\pm - \alpha_0^\pm \bar{\phi} \frac{ck_z (1 + \tau)}{A_0 \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega} \right],
\]  

(33)

\[
\frac{\hat{A}^{(1)}_\pm}{D^{(1)}} \frac{\hat{A}^{(0)}_\pm}{D^{(0)}} - \frac{\alpha_i^\pm}{D^{(0)}} \left[ 1 - \alpha_0 \bar{\phi}_0 \frac{ck_z (1 + \tau)}{A_0 \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega} \right].
\]  

(34)

We are able to transform in the same manner either the expression (20) for electrostatic potential:

\[
\hat{\phi}_\pm = \hat{\phi}^{(0)}_\pm + \hat{\phi}^{(1)}_\pm,
\]  

(35)

where

\[
\hat{\phi}^{(0)}_\pm = \mp \frac{ck_z T_{ik} k_\perp^2}{4 \pi e^2 n_0 D^{(0)}} \left[ \alpha_i^\pm + \alpha_2^\pm - \alpha_0^\pm \bar{\phi}_0 \frac{ck_z (1 + \tau)}{A_0 \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega} \right],
\]  

(36)

\[
\hat{\phi}^{(1)}_\pm = \mp \frac{D^{(1)}}{D^{(0)}} \frac{\hat{\phi}^{(0)}_\pm}{D^{(0)}} - \frac{ck_z T_{ik} k_\perp^2}{4 \pi e^2 n_0 D^{(0)}} \left[ \alpha_i^\pm \left( \frac{1 + k_\perp^2 \lambda_s^2 \Omega \omega_k - \omega_\omega} \right) + \\
2 \alpha_0^\pm \frac{k_x q_x}{k_\perp^2} \left( 1 - \alpha_0 \bar{\phi}_0 \frac{ck_z (1 + \tau)}{A_0 \left( 1 + k_\perp^2 \lambda_s^2 \right) \omega_k - \omega_\omega} \right) \right].
\]  

(37)

Now, looking to relation (28), we find that \( \hat{\delta}^{(0)}_\pm = 0 \). This equation describes the remarkable fact that the main contribution in the evolution equation of the average magnetic field of the “magnetic” and “electrostatic” parts of amplitudes of the secondary – satellite small scale modes relatively decreases (see the equations (23)-(27)). Consequently, for \( \hat{\delta}_\pm \) we have the decomposition:

\[\uparrow\]

The first author is deceased
\[ \hat{\delta}_\pm^{(1)} = -\frac{1}{D^{(0)}} \left\{ \alpha^\pm_1 \left[ 1 - \alpha_0 \frac{\alpha^c_0}{A_0} \left( \frac{ck_z(1 + \tau)}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} \right) \right] \left\{ \Omega - \frac{2k_q q_x}{k_{\perp}} (\omega_k - \omega_1) \right\} - \right. \\
\alpha^\pm_1 \frac{\omega_k - \omega_e}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} \right\}, \]
\[ \hat{\delta}_\pm^{(2)} = \pm \frac{\Omega}{D^{(0)2}} \left[ \alpha^\pm_1 \left[ 1 - \alpha_0 \frac{\alpha^c_0}{A_0} \left( \frac{ck_z(1 + \tau)}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} \right) \right] \left\{ D^{(1)} + (\omega_k - \omega_1) \frac{q_x}{k_{\perp}} \times \right\} \\
q_x \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right] - \left\{ (1 + k^2_{\perp} \lambda^2_s) \omega_1 - \omega_e \right\} - 2k_q \Omega \left[ (1 + k^2_{\perp} \lambda^2_s) \right] \right\} - \right. \\
\left. \alpha^\pm_1 \frac{\omega_k - \omega_e}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} D^{(1)} \right\}. \]

Let's mention that the term \( \hat{\delta}_\pm^{(1)} \) does not make any contribution into the expression (26) for \( R_2 \). Thus, the relation (26) gets the form:
\[ R_2 = \tilde{\phi} \cdot \hat{\delta}_+^{(2)} - \tilde{\phi} \cdot \hat{\delta}_-^{(2)}. \]

3.4. Expression for amplitudes of the large scale modes

Using (29)-(41) and (19) the expressions for \( R_1 \), \( R_2 \) and \( R_3 \) (25)-(27) can be lead to: \( R_1 \)
\[ R_1 (k) = \frac{ic^2 (1 + \tau) q_x k_y k_x \omega \Omega}{B_0 D^{(0)2} \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right]} I_k \left[ R_1^A A_0 + \frac{ck_z(1 + \tau)}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} R_1^\phi \phi_0 \right], \]
\[ R_2 (k) = \frac{ic q_x k_y \Omega}{B_0 D^{(0)2} \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right]} I_k \left[ R_2^A A_0 - \frac{ck_z(1 + \tau)}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} R_2^\phi \phi_0 \right], \]
and
\[ R_3 (k) = \frac{ic q_x k_y (1 + \tau)}{B_0 D^{(0)2} \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right]} I_k \left[ R_3^A A_0 - \frac{ck_z(1 + \tau + k^2_{\perp} \lambda^2_s)}{(1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e} R_3^\phi \phi_0 \right]; \]

Here
\[ R_1^A = 2(1 + \tau) \left[ \omega_k - \left( (1 + k^2_{\perp} \lambda^2_s) \omega_e \right) \right] \left[ \Omega - \frac{q_x}{k_x} (\omega_k - \omega_1) b_1^A \right], \]
\[ R_1^\phi = \left( (1 + \tau + k^2_{\perp} \lambda^2_s) \left[ 2 \Omega \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right] - \frac{q_x}{k_x} (\omega_k - \omega_1) b_1^\phi \right] \right], \]
\[ b_1^A = \frac{\left( k^2_x - k^2_y \right) \omega_e - \left( (1 + k^2_{\perp} \lambda^2_s) \omega_k + 2(1 + k^2_{\perp} \lambda^2_s) k^2_x \omega_k \right)}{k^2_{\perp} \omega_e - \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k \right]} \]
\[ b_1^\phi = \frac{1}{k^2_{\perp}} \left[ 2 \left( (1 + k^2_{\perp} \lambda^2_s) k^2_{\perp} - k^2_y \right) \omega_k + \left( (1 + k^2_{\perp} \lambda^2_s) k^2_y \omega_n + 3 k^2_x - k^2_y \right) \omega_e \right]; \]
\[ R_2^A = \left( (1 + \tau + k^2_{\perp} \lambda^2_s) \left[ \Omega^2 - 2 \frac{k_q q_x}{k_{\perp}} \Omega (\omega_k - \omega_1) + \frac{q_x^2}{k_{\perp}^2} (\omega_k - \omega_1)^2 \right] \right), \]
\[ R_2^\phi = \left( (1 + \tau + q^2_{\perp} \lambda^2_s) \left[ \Omega^2 - \frac{q_x}{k_{\perp}} \Omega (\omega_k - \omega_1) \right] \right), \]
\[ R_3^A = \left( (1 + \tau + q^2_{\perp} \lambda^2_s) \left[ \Omega^2 \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right] \right] \right), \]
\[ R_3^\phi = \left( (1 + \tau + q^2_{\perp} \lambda^2_s) \left[ \Omega^2 \left[ (1 + k^2_{\perp} \lambda^2_s) \omega_k - \omega_e \right] \right] \right). \]

† The first author is deceased.
2Ω \frac{k x q_x}{k_1^2} (\omega - \omega_i) \left[ (1 + k_1^2 \lambda_s^2) + 2 \frac{q_x^2}{k_2^2} \omega_i - \omega_i \right] + 2 \frac{q_x^2}{k_2^2} (\omega - \omega_i)^2 \left[ (1 + k_2^2 \lambda_s^2) / 2 \right];

\text{(50)}

R_A^3 = 2k_x \Omega [\omega - (1 + k_1^2 \lambda_s^2) \omega_i] - \frac{q_x}{k_x k_1} (\omega - \omega_i) \left[ (k_1^2 - k_2^2) \omega_i - \left( (1 + k_1^2 \lambda_s^2) k_1^2 \omega_i + (1 + k_2^2 \lambda_s^2) k_2^2 \omega_i \right) \right],

\text{(51)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(52)}

\text{where}

l_1^6 = (1 + \tau) \frac{q_x k_x}{k_1^2} \left[ (\omega - \omega_i) \Gamma_0^2 \right],

\text{(53)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(54)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(55)}

\text{where}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(56)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(57)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(58)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(59)}

\text{and the functions } \left( l_1^6, l_1^6, l_1^6, l_1^6 \right) \text{ denote}

\text{(60)}

\text{where}

\text{(61)}

\text{From the closed system of equations (54) and (55), we simply get the dispersion equation for}

l_1^6 = - (1 + \tau) \left[ \frac{k x \Gamma_0^2 \left( R_1^6 + \frac{q x \lambda^2}{\Omega} R_A^3 \right)}{2 (1 + k_2^2 \lambda_s^2) \omega - \omega_i} \right],

\text{(62)}

\text{large scale zonal flows and the magnetic fields:}

\text{1 - } \left( l_1^6 + l_1^6 \right) + l_1^6 l_1^6 = 0,

\text{(63)}

\text{The first author is deceased}
The dispersion relation of the zonal modes (63) allows an investigation of their generation via continuous spectrum of the initial modes with skin scale, which is the main subject of the traditional theory of such generation, increasing to a kinetic equation for the waves, summarized in (Diamond, et al., 2005). Thus, the approach developed in section 3.4, based on dynamic equations of magnetic hydrodynamics of the ionosphere, is an alternative to the approach in (Diamond, et al., 2005) and in our opinion, is more convenient in to realize, also in the interpretation of results obtained based on them. It’s obvious that the dispersion relation (63) represents bisquared equation according to \( \Omega - q, V_g \). This equation can be reduced to a squared one for a very interesting range of frequencies \( \Omega \) of the zonal perturbation.

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ზონალური დინებებისა და მაგნიტური ველების გენერაციის
უძველო თხზულებიდან მხიდარნიშნული გარემოს საქმევალურო

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რეზიუმე

აღნიშნულ ნაშრომში შესწავლილია დიდმასშტაბიანი ზონალური დინებისა და მაგნიტური ველის გენერაცია მოკლეტალღოვანი არადაჯახებ ადი სკინ სისქის დრეიფული ალფენის ტურბულენტობით იონოსფეროში. მიღებულ იყო სკინ სისქის ტალღური სტრუქტურების აღმწერი თვითშეთანხმებული ორ მ ოდელური არაწრფივ განტოლებისგან შემდგარი სისტემა. მიღებულია არაერთგვარ წანაცვლებითი დინებისა და მაგნიტური ველების ევოლუციის განტოლება ჩქარი მაღალსიხშირული და მოკლეტალღოვანი ფლუქტუაციების აღმწერი მოდელური განტო ლებების გასაშუალებით მულტიმასშტაბური გაშლის საფუძველზე. ჩარგაზია, რომ აქტიური ღონისძიებებისა და მაგნიტური ველის ღონისძიების შემდგომ, საწყისი ტალღების სპექტრის ზრდისთან ერთად, არამდგრადობა იკლებს.

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Изучение генераций зональных течений и магнитных полей в ионосфере на основе мультинародного представления

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Резюме

В настоящей работе изучается генерация крупномасштабных зонального течения и магнитного поля коротковолновьым безсталкновительным Альвеновской турбулентностью порядка толщины электронного скин слоя в ионосфере. Получена самосогласованная система двух модельных нелинейных динамических уравнений, описывающая динамику волновых структур с характерическим масштабом скин размера. Выведены уравнения эволюции неоднородного сдвигового течения и магнитного поля усреднением модельных уравнений для быстрых високочастотных и короткомасштабных флуктуаций на основе мультинародного разложения. Показано, что крупномасштабные возмущения плаэмы течения и магнитного поля коротковолновьым безсталкновительным Альвеновской турбулентностью нелниейным воздействием напряжении Рейнольдса и Максвелла. Положительная обратная связь в системе достигается модуляцией волн скинового размера крупномасштабным зональным течением и/или возбуждёнными крупномасштабными магнитными полями. В результате, распространение короткомасштабных волновых пакетов в ионосфере сопутствуется низкочастотными, длинноволновыми возмущениями, нелинейным воздействием неустойчивостью. Изучены два типа режима неустойчивостей – резонансно-станавлинические и гидродинамические. Также найдены инкременты неустойчивости. Определены условия развития неустойчивости и возможность генерации крупномасштабных структур. Нелинейный инкремент этих взаимодействий значительно зависит от волнового вектора Альвеновской накачки и от характеристического размера генерированной зональной структуры. Это означает что неустойчивость перекачивает энергию значительно коротковолновых Альвеновских волн в крупномасштабных зональных структур, которые характерны для обратного каскада турбулентности. Инкремент энергии накачки значительно зависит от ширины спектра волн накачки и при увеличении ширины спектра начальных волн, устойчивость может быть уменьшено.

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